

**St. Mary's V.A. C.E. Primary School**  
**Progression in the four rules of calculation**

n.b. All highlighted notes apply to staff only.

- This policy is intended to aid all staff and parents in their understanding of the four rules and to provide progression in written calculations.
- It builds on the approach from the National Numeracy Strategy Framework.
- **The policy guidance should be referred to when planning, in conjunction with the National Numeracy Strategy.**
- Written methods of recording will be used once children have secure mental number knowledge and skills.
- The policy is to give guidance regarding progression, which the majority of pupils will make.
- Practical equipment to demonstrate should be normal practice in all age groups and particularly with younger or less able pupils.
- Always build from one model of recording to another with both methods shown side by side.
- Teachers need to be flexible in their approach and recognise that some children will be ready to progress to the next stage, whilst others will need consolidation of previous stages no matter their age or year group.
- It is far better for children to be able to operate effectively at any stage, with understanding, than to move them on too quickly.
- **Not all children will reach a formal written method in each of the four rules by the end of year 6.**
- It is better to achieve successful levels of accuracy using a number line than poor understanding of the standard algorithm.
- Children will be encouraged to use their own effective method to perform calculations.

### **Teaching Points**

When approaching a calculation, children should be encouraged to ask themselves the following questions . . . .

- Can I do this in my head?
- Do I know the approximate size of the answer? ( Estimation)
- Do I need to use a calculator to work this out?
- If I can't answer it wholly in my head, what do I need to write down in order to help me calculate the answer? (jottings)
- Which written method would be helpful?

Whenever appropriate, children should do a mental calculation. For example which of these would you do mentally?

$$3002 - 2998 =$$

$$9563 - 3771 =$$

In order to encourage mental calculations strategies, calculations should always be presented to children horizontally so that they can decide how to tackle them.

Ensure when using textbooks that questions are presented appropriately and that explanations of methods follow the school policy.

If using textbooks where the method does not match our school policy provide pupils with the correct exemplification.

When modelling calculations use language that reflects the size of the numbers involved. (E.g. carry ten rather than carry one).

### Overview of Methods

- It is expected that addition/subtraction and multiplication/division be taught alongside each other so that pupils can see the inverse relationship between them.
- Pupils should be taught to estimate their answers first and check calculations with a variety of strategies including the inverse operation.
- It is better to do fewer calculations and do the checks than to do pages of calculations
- This progression for all calculation processes is to illustrate the methods of recording rather than the size of the numbers.
- Whilst later stages make explicit reference to application to problem solving contexts these should be used throughout at the appropriate level for the pupils concerned.

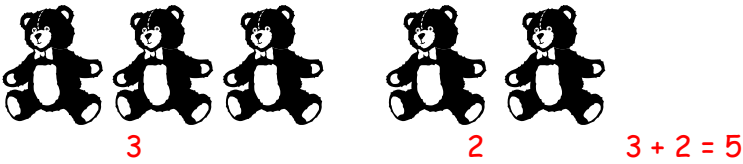

**Further details can also be found in**

'Teaching Written Calculations: Guidance for teachers at KS1 and KS2'  
QCA /99/486 tel: 01787 884444

**New Primary Strategy Site on the Government Archive Site**

**Original Numeracy Strategy Folder..... Written methods are section 5 p45 and section 6 p 50**

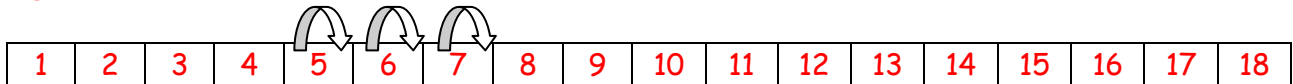
### Progression In Written Methods For Addition

Question	Typical Recording
<p><b>Stage 1</b> Counting, knowing the order of numbers to 5, then 10, lots of practical activities with no written recording.</p> <p>A. One-to-one correspondence and number retention in order to fully understand the concepts of addition.</p> <p>B. Counting forwards in 1's</p> <p>C. Finding 1 more than.</p> <p>D. Combining 2 groups of objects and begin to record pictorially (or numbers).</p> <p>E. Finding totals. Separating a total number of objects into 2 groups. Develop through into the various ways this can be done. ie; 6 objects can be partitioned into 3 and 3, 4 and 2 etc.</p> <p>F. How many more are needed? ie; I have 5 cakes and 7 children want one. How many more cakes do I need?</p> <p>G. Understand the way of writing down addition. Introduce + and = signs. Begin to relate all of the above to the written version.</p> <p>H. Adding by counting on. E.g. Children to start counting from 5 onwards. Here are 5 cakes, what would 3 more cakes total?</p> <p>I. Adding 0 always leaves a number unchanged.</p>	
<p>Jane has 3 bears. She was given 2 more. How many does she have now?</p>	 <p>Children will use a mixture of words and symbols in order to explain to someone else the methods they have used.</p>
<p>There are 5 children sitting on the carpet and one more joins them. How many children are on the carpet now?</p>	 <p>6 counters represent objects</p>

**Stage 2 Stage 2**

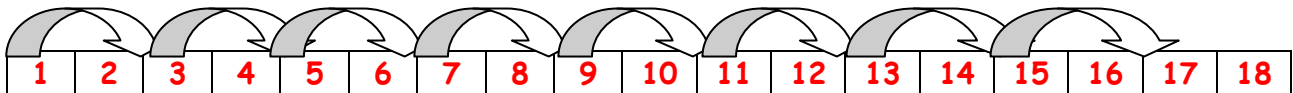
The use of numbered number tracks and lines is very helpful for teaching children the order of numbers and for images of addition and subtraction. It may begin with children physically jumping forwards and backwards along a numbered number track.

E.g.  $5 + 3$



Children can then use the track for finding patterns.

E.g., Mark the numbers you land on when you hop forward in twos from different starting numbers.



The following methods represent how the children might record their mental methods. There will be many variations:

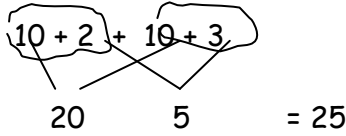
**Stage 3**

Building on mental methods using a number line and then an empty number line. The empty number line helps to record the steps on the way to calculating the total.

<p><math>12 + 3</math> <math>8 + 7 = 15</math></p> <p>Building on mental methods using a number line. First with numbers. Then empty line.</p>	
<p><math>13 + 8</math></p> <p>Adding to the next whole ten and then the 'extra'</p>	
<p><math>23 + 15 = 38</math></p> <p>The mental methods that lead to column addition may involve partitioning, e.g. adding the tens and units separately, often starting with the tens.</p>	<p>This mental method of partitioning is the basis of all column written methods of addition.</p>

**Stage 4**

Once children are secure with mentally partitioning numbers, record mental methods using partitioning. Add the tens and then the units to form partial sums and then add these partial sums.

<p>12 + 13</p>	<p><i>This mental method of partitioning is the basis of all column written methods of addition. Model with Diennes regularly.</i></p> 
<p>93 + 76</p>	<p><i>Model with Diennes regularly.</i></p> <p>= 90 + 70 + 3 + 6                  = 90 + 70 + 9                  = 160 + 9                  = 169</p>

*Do not introduce column addition if children are not conversant with partitioning.*

**Stage 5**

Move on to a layout showing the addition of the tens to the tens and the units to the units separately. To find the partial sum children should be taught to add the units digits first.

<p>93 + 76</p>	<p><i>Model with Diennes regularly. Refer to 90 + 70..9 tens add 7 tens</i></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>H</td><td>T</td><td>U</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td>9</td><td>3</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>+</td><td></td><td>7</td><td>6</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td>9</td><td>=</td><td>3</td><td>+</td><td></td><td>6</td><td></td></tr> <tr><td></td><td>1</td><td>6</td><td>0</td><td>=</td><td>9</td><td>0</td><td>+</td><td>7</td><td>0</td></tr> <tr><td></td><td>1</td><td>6</td><td>9</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>		H	T	U									9	3							+		7	6										9	=	3	+		6			1	6	0	=	9	0	+	7	0		1	6	9																										
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<p>438 + 275</p> <p>This expanded method will lead to the more compact method so that they can understand its structure and efficiency.</p>	<p><i>Model with Diennes regularly. Refer to 30+ 70..3 tens add 7 tens                  400 + 200 .. 4 hundreds add 2 hundreds.</i></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>H</td><td>T</td><td>U</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>4</td><td>3</td><td>8</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>+</td><td>2</td><td>7</td><td>5</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td>1</td><td>3</td><td>=</td><td></td><td>8</td><td>+</td><td></td><td></td><td>5</td></tr> <tr><td></td><td>1</td><td>0</td><td>0</td><td>=</td><td></td><td>3</td><td>0</td><td>+</td><td></td><td>7</td><td>0</td></tr> <tr><td></td><td>6</td><td>0</td><td>0</td><td>=</td><td>4</td><td>0</td><td>0</td><td>+</td><td>2</td><td>0</td><td>0</td></tr> <tr><td></td><td>7</td><td>1</td><td>3</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>		H	T	U									4	3	8								+	2	7	5										1	3	=		8	+			5		1	0	0	=		3	0	+		7	0		6	0	0	=	4	0	0	+	2	0	0		7	1	3								
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**Stage 6**

In this compact, column method, recording is reduced further. Carry digits are recorded below the line, using the phrases 'carry ten' or 'carry one hundred' not carry one. This method can be applied to numbers with varying numbers of digits.

$$438 + 275$$

	H	T	U							
	4	3	8							
+	2	7	5							
	7	1	3							
	1	1								

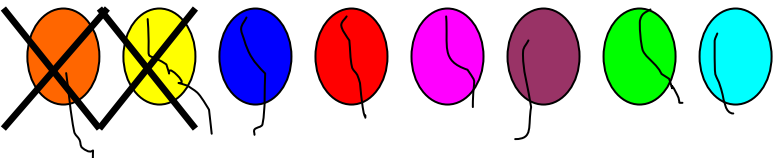
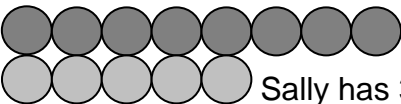
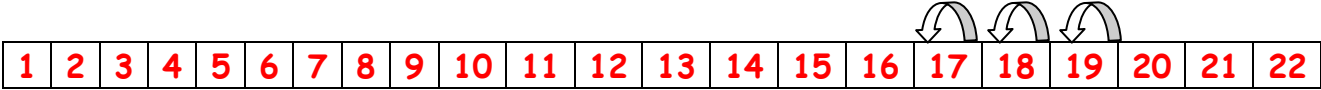
Still use the language of hundreds, tens and units.

**Stage 7 - Using and applying addition methods in a problem solving context and to decimal numbers. (EXTENSION)**

E.g. Find the total weight of 5 adults weighing 72kg, 57.4kg, 89.75kg, 72.9kg and 89.4kg to determine if they can all get in a lift with a total weight restriction of 400kg.

$$\begin{array}{r}
 72.00 \\
 57.40 \\
 89.75 \\
 72.90 \\
 + 89.40 \\
 \hline
 381.45 \\
 32
 \end{array}$$

Progression In Written Methods For Subtraction

Question	Typical Recording
<p><b>Stage 1</b>                      Counting backwards, knowing the order of numbers to 5 , then 10, lots of practical activities with practical recording, no written recording.</p> <p>Children need practical activities of taking away, that is finding how many are left from a collection of objects when some are removed.</p> <p>A mixture of words and symbols will be used by children in order to explain to someone else the methods that they have used. Children will use a variety of ways of recording subtraction, reflecting the mental methods used.</p>	
<p>There were 8 balloons.                      Two popped. How many are left?                      Take away/ Subtraction</p>	 <p><math>8 - 2 = 6</math></p> <p>Pupils draw pictorially</p>
<p>How many more biscuits does Sally have than you?                      Children also need practical activities around 'finding the difference', which involves making a comparison between the numbers in two groups of objects. They need to recognise that this is another example of subtraction.                      Difference</p>	 <p>Sally has 3 more than me.</p> <p>Pupils can use role play materials, counters etc.</p>
<p><i>The following methods represent how the children might record their mental methods. There will be many variations:</i></p>	
<p><b>Stage 2</b>                      The use of numbered number tracks and lines is very helpful for teaching children the order of numbers and for images of addition and subtraction. It may begin with children physically jumping forwards and backwards along a numbered number track.</p> <p><i>Eg, There are 20 children in our class. Three are away today. How many are here?</i></p>  <p><math>20 - 3 = 17</math></p>	

**Stage 3**

The empty number line helps to record or explain the steps in mental subtraction. Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10

Some children prefer to count on from the smaller to the larger number to find the difference. Particularly mentally this may be a more efficient method. For example consider whether you would count up or back when solving these calculations.

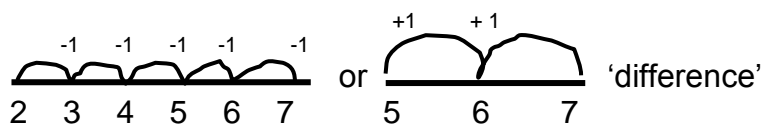
57 - 12

86 - 77

43 - 28

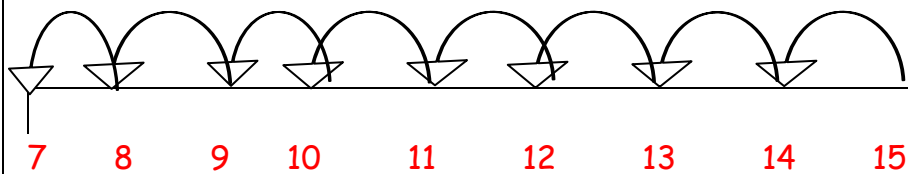
Each of the methods explained in points (i) - (iv) can be applied to this counting on method as explained in these examples:

7 - 5

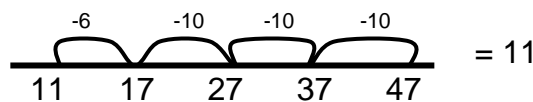


15 - 8

(i) Counting back in units/ counting on in units

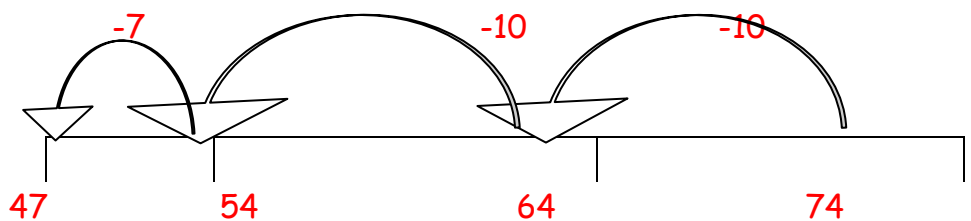


47 - 36

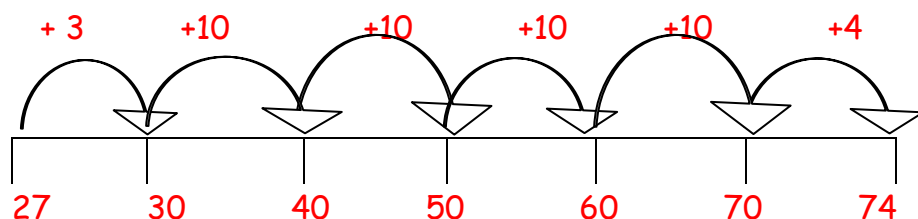


74 - 27

(ii) Counting back/ on by partitioning the number into tens and units



OR: If Finding the Difference





<p><math>74 - 27 =</math></p> <p>(iii) With practice children will need to record less information</p>	<p><math>74 - 27 = 74 - 20 - 7 = 54 - 7 = 47</math></p> <p>OR</p> <p>E.g. <math>84 - 56</math></p>
<p><math>30 - 9</math>  <math>(30 - 10) + 1 =</math></p> <p>(iv) Children may use efficient mental methods to inform their written methods, e.g. adjusting</p>	

<b>Stage 4</b>		
Building on the number line, children partition and subtract mentally. First children should be given sums where no decomposition is required.		
$47 - 36$	$47 - 36 = 47 - (30 + 6) = 11$ $47 - 30 = 17$ $17 - 6 = 11$ So $47 - 36 = 11$ <b><i>N.B. The format below is not mathematically correct</i></b> $47 - 36 = 47 - (30 + 6) = 47 - 30 = 17 - 6 = 11$	
$47 - 36$	<i>Model with Diennes regularly.</i> $= (40 \text{ and } 7) - (30 \text{ and } 6)$ $= 40 - 30 \text{ and } 7 - 6 \text{ is } 1$ $= 10 + 1$ $= 11$	
Then children can cross the tens boundary $75 - 47$	$75 - 47 = 75 - (40 + 7) = 28$ $75 - 40 = 35$ $35 - 7 = 28$ So $75 - 47 = 28$ <b><i>N.B. The format below is not mathematically correct</i></b> $75 - 47 = 75 - (40 + 7) = 75 - 40 = 35 - 7 = 28$	

*Do not introduce decomposition if children are not fully competent in partitioning numbers.*

**Stage 5**

Stage 5 is the expanded layout, which leads to the column method. Partitioning the numbers into tens and units and writing one under the other, mirrors the column method where tens are placed under tens and units under units. This method parallels the column addition method.

This can be applied to 3 or 4 digit numbers.

This method is written in the initial stages, but children should be encouraged to tackle the stages mentally when confident.

<p>47 – 36</p> <p>74 – 23</p> <p>This method can be applied to decomposition see below</p>	<p><i>Model with Diennes regularly.</i></p> <table border="1" style="width: 100%; text-align: center;"> <tr><td></td><td>T</td><td></td><td></td><td></td><td>U</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>4</td><td>0</td><td></td><td>+</td><td>7</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>-</td><td>3</td><td>0</td><td></td><td>+</td><td>6</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>1</td><td>0</td><td></td><td>+</td><td>1</td><td>=</td><td>1</td><td>1</td><td></td><td></td><td></td></tr> </table> <table border="1" style="width: 100%; text-align: center;"> <tr><td></td><td>T</td><td></td><td></td><td></td><td>U</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>7</td><td>0</td><td></td><td>+</td><td>4</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>-</td><td>2</td><td>0</td><td></td><td>+</td><td>3</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>4</td><td>0</td><td></td><td>+</td><td>1</td><td>=</td><td>4</td><td>1</td><td></td><td></td><td></td></tr> </table>		T				U								4	0		+	7							-	3	0		+	6								1	0		+	1	=	1	1					T				U								7	0		+	4							-	2	0		+	3								4	0		+	1	=	4	1			
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**Stage 6**

This method leads to a more compact method.

This can be applied to 2,3 or 4 digit numbers.

It should not be the automatic method chosen; pupils should be encouraged to use the strategy most appropriate to the calculation given.

$$754 - 665$$

*Model with Diennes regularly.*

$$\begin{array}{r} \overset{6}{7} \overset{14}{5} \overset{1}{4} \\ - 665 \\ \hline 89 \end{array}$$

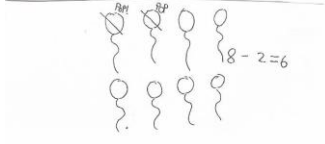
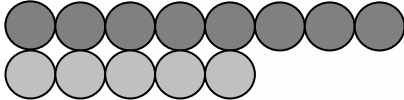



Still use the language of tens and hundreds.

**Stage 7 - Using and applying subtraction methods in a problem solving context and to decimal numbers. (EXTENSION)**


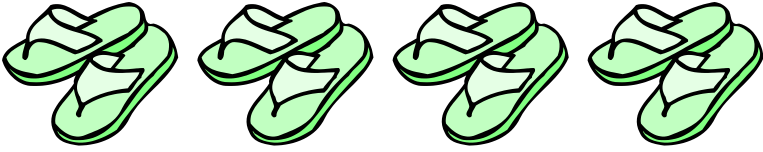

E.g. Find the difference in weight of 2 adults weighing 57.8kg, and 89.75kg.

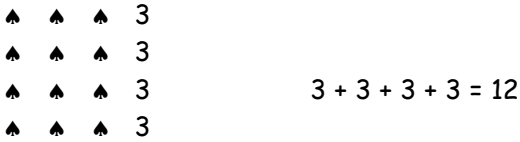
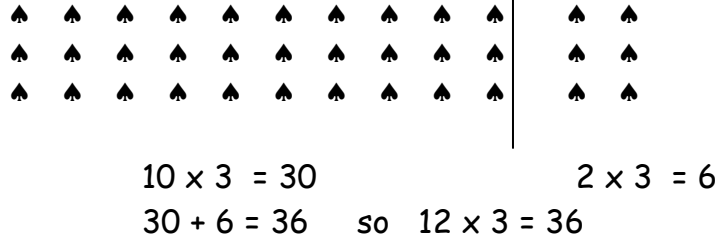
$$\begin{array}{r} \phantom{8} \overset{8}{9} \overset{17}{7} \overset{5}{5} \\ - 57.80 \\ \hline 31.95 \end{array}$$

Progression In Written Methods For Subtraction – Counting Up

Question	Typical Recording	Year
Counting backwards, knowing the order of numbers to 5, then 10 Lots of practical activities with practical recording, no written recording		
There were 8 balloons. Two popped. How many are left?		
How many more biscuits does Sally have than you?		than me.
<i>The following methods represent There will be many variations:</i>		<i>d their mental methods.</i>
7 – 5		
47 – 36		
47 – 36		
<b><i>Do not introduce decomposition if children are not fully competent in partitioning numbers.</i></b>		
75 - 47	<p><i>Model with Diennes regularly.</i></p> $\begin{array}{r} 75 \\ - 47 \\ \hline 3 \quad \text{to make } 50 \\ 20 \quad \text{to make } 70 \\ 5 \quad \text{to make } 75 \\ \hline 28 \end{array}$	
754 – 635	<p><i>Model with Diennes regularly.</i></p> $\begin{array}{r} 754 \\ - 635 \\ \hline 5 \rightarrow 640 \\ 60 \rightarrow 700 \\ 54 \rightarrow 754 \\ \hline 119 \end{array}$	

Progression In Written Methods For Multiplication

Question	Typical Recording																					
<p><b>Stage 1</b></p> <p>It is expected that there will be lots of practical activities with practical recording, no written recording, to support children's growing awareness and understanding of multiplication.</p> <p>Children can complete practical activities involving grouping objects. Rhymes and stories can be used that involve counting in different intervals.</p> <p>Counting in ones, tens, twos...e.g. jumping along a number line</p> <p>Use apparatus to sort objects into groups. Early foundations of arrays.</p> <p>E.g. Sort six compare bears into 2 groups. How many in each group?</p> <div style="display: flex; justify-content: space-around; align-items: center;">  </div> <p>2 lots of 3 2 groups of 3 <math>3 \times 2</math></p> <p>A mixture of pictures, words and symbols will be used by children in order to explain to someone else the methods that they have used.</p>																						
<p>Find how many shoes using counting in twos for speed</p>																						
<p>Count in 2's</p>	<p>2   4   <input type="text"/>   8</p>																					
<p><b>Stage 2</b></p> <p>Children will begin to recognise multiplication as repeated addition.</p> <p>What is the value of 4 five-pence coins?</p> <p><math>5p + 5p + 5p + 5p</math> 4 groups of 5p <math>5p \times 4</math></p> <div style="display: flex; justify-content: space-around; align-items: center;"> <p>+5</p> <p>+5</p> <p>+5</p> <p>+5</p> </div>  <table border="1" style="width: 100%; text-align: center;"> <tr> <td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td> </tr> </table>		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		

<p>Stage 2 cont.                  Show <math>3 \times 4</math>                  (as an array)                  There should be lots of work on arrays using multilink etc before dot patterns</p>	
<p><math>12 \times 3</math></p>	<p>Model with multilink regularly</p> 
<p><math>X \div 10</math> or 100</p>	<p><math>700 \longrightarrow 7</math></p>
<p><b>Stage 3</b>                  This method of mental multiplication using partitioning allows the tens and units to be multiplied separately to form partial products and these are then added to find the total product.                  To be able to use written methods of multiplication successfully it is important children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication. For example:</p> <ul style="list-style-type: none"> <li>* recalling multiplication facts.</li> <li>* multiplying by 10 and 100.</li> <li>* partitioning numbers into multiples of hundreds, tens and one.</li> <li>* add two or more single digit numbers mentally.</li> <li>* add multiples of 10 or 100.</li> <li>* add combinations of whole numbers using the column method.</li> </ul>	
<p><math>43 \times 4 =</math></p>	<p>Model with Diennes regularly</p> $\begin{array}{r} 40 + 3 \\ \times 4 \\ \hline 160 \quad 12 = 172 \end{array}$
<p><math>12 \times 3</math>  (partitioning)  <math>43 \times 4 =</math></p>	<p>Model with Diennes regularly</p> $\begin{aligned} &= (10 + 2) \times 3 \\ &= (10 \times 3) + (2 \times 3) \\ &= 30 + 6 \\ &= 36 \end{aligned}$ <p><i>(not necessarily recorded in exactly this way)</i></p> $\begin{aligned} &= (40 + 3) \times 4 \\ &= (40 \times 4) + (3 \times 4) \\ &= 160 + 12 = 172 \end{aligned}$
<p><math>12 \times 30</math> (factorising)                  Some children find this a very confusing method                  Class teacher to assess suitability for class.</p>	<p>Model with Diennes regularly</p> $\begin{aligned} &= 12 \times (3 \times 10) \\ &= (12 \times 3) \times 10 \\ &= 36 \times 10 \\ &= 360 \end{aligned}$

**Stage 4**

The grid method is the main method to be taught and generally produces the least amount of errors.

*The grid method and the standard method rely on understanding the partitioning method of multiplication*

12 x 3	<i>Model with Diennes</i>			
		T	U	
	x	10	2	
	3	30	6	= 36

12 x 72  Use the language of tens and units throughout.		T	U	
	x	10	2	
	70	700	140	= 840
	2	20	4	= 24
		720	144	= 864

**Stage 5**

Extend the grid method to HTU x TU

286 x 29	x	200	80	6	
	20	4000	1600	120	5720
	9	1800	720	54	2574
		5800	2320	174	= 8294

**Stage 6**

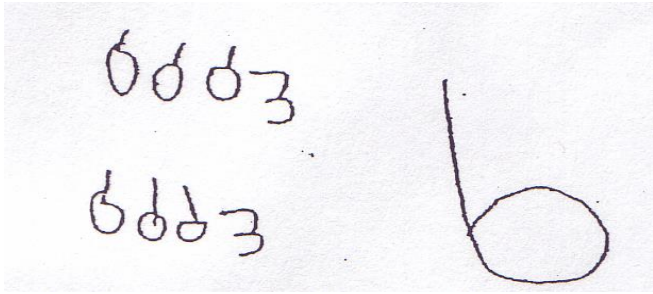
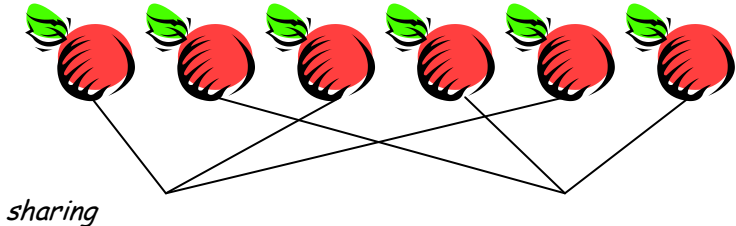
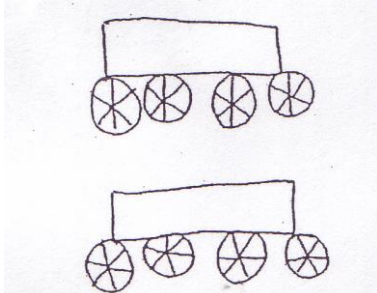
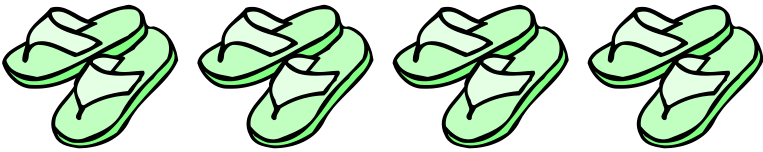
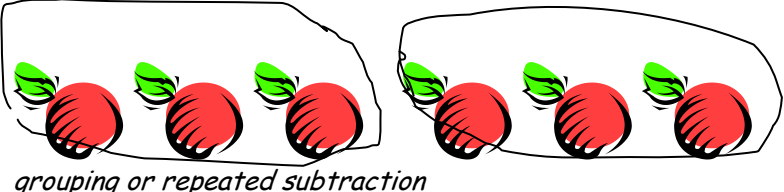
The grid method can be extended to bigger numbers and decimals  
 This can be presented vertically however the grid method is the most straightforward and efficient way of multiplying and would allow children to tackle any multiplication problem.  
 Extended column method should at first be introduces when the grid method is secure and demonstrated in a parallel manner.

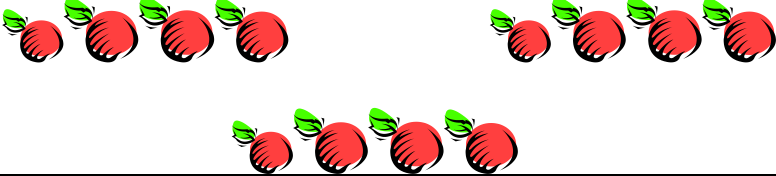
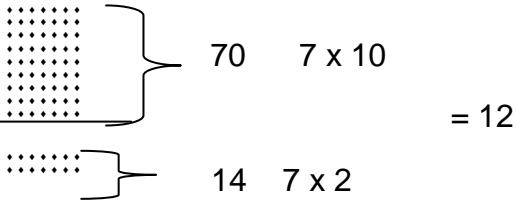
12 x 3  Use the language of tens and units throughout.		T	U					
		1	2					
	x		3					
		3	0	=	1	0	x	3
			6	=		2	x	3
		3	6					



<p><math>12 \times 3</math> Use the language of tens and units throughout.</p>	<table border="1"> <tr><td></td><td>T</td><td>U</td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td>1</td><td>2</td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>x</td><td></td><td>3</td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td>6</td><td>=</td><td>2</td><td>x</td><td>3</td><td></td></tr> <tr><td></td><td>3</td><td>0</td><td>=</td><td>1</td><td>0</td><td>x</td><td>3</td></tr> <tr><td></td><td>3</td><td>6</td><td></td><td></td><td></td><td></td><td></td></tr> </table>		T	U							1	2						x		3								6	=	2	x	3			3	0	=	1	0	x	3		3	6																																					
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<p><math>12 \times 72</math> Use the language of hundreds, tens and units throughout.</p>	<table border="1"> <tr><td></td><td>H</td><td>T</td><td>U</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td>7</td><td>2</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>x</td><td></td><td>1</td><td>2</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td>4</td><td>=</td><td>2</td><td>x</td><td>2</td><td></td><td></td></tr> <tr><td></td><td>1</td><td>4</td><td>0</td><td>=</td><td>7</td><td>0</td><td>x</td><td>2</td><td></td></tr> <tr><td></td><td></td><td>2</td><td>0</td><td>=</td><td>2</td><td>x</td><td>1</td><td>0</td><td></td></tr> <tr><td></td><td>7</td><td>0</td><td>0</td><td>=</td><td>7</td><td>0</td><td>x</td><td>1</td><td>0</td></tr> <tr><td></td><td>8</td><td>6</td><td>4</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>		H	T	U									7	2							x		1	2										4	=	2	x	2				1	4	0	=	7	0	x	2				2	0	=	2	x	1	0			7	0	0	=	7	0	x	1	0		8	6	4						
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<p><math>56 \times 27</math> Calculations involving carrying</p>	<p> <math display="block">\begin{array}{r} 56 \\ \times 27 \\ \hline 42 \\ 350 \\ 120 \\ \hline 1000 \\ \underline{1512} \\ 1 \end{array}</math> </p> <p>(<math>6 \times 7 = 42</math>)  (<math>50 \times 7 = 350</math>)  (<math>6 \times 20 = 120</math>)  (<math>50 \times 20 = 1000</math>)</p> <p>This moves on to ...</p> <p> <math display="block">\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \\ 1120 \\ \hline 1512 \\ 1 \end{array}</math> </p>																																																																																

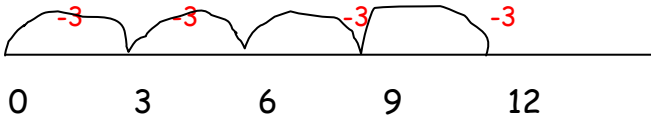
Progression In Written Methods For Division

Question	Typical Recording
<p><b>Stage 1</b>                      Counting, knowing the order of numbers to 5, then 10                      Lots of practical activities with practical recording, no written recording                      Sharing out toast between pupils on the table etc.                      Young children will be familiar with the language of sharing and understand that six shared equally among three people means everyone has two each and that if they were shared between two people, both would have three.</p>	
<p>6 cherries shared equally between 2 people.                      How many cherries does each one get?                      Sharing</p>	
<p><b>Stage 2</b>                      Children can draw pictures to explain to someone else how they have solved a simple division problem.                      How many cars can you make with 4 wheels each if you have eight wheels?</p>	
<p>There are 8 shoes altogether. How many pairs of shoes are there?                      Chunking/ Grouping</p>	
<p>There are 6 cherries in a bag. How many people can get 3 cherries each?                      Chunking/Grouping</p>	

<p><i>The following methods represent how the children might record their mental methods. There will be many variations and links with multiplication: THEY ARE NOT ALL TO BE DONE BEFORE STAGE 3!!!!!!</i></p>													
<p>12 ÷ 3</p> <p>Sharing</p>													
<p>12 ÷ 3</p> <p>Chunking/ Grouping link to xn arrays</p> <p>3 for free idea</p>	<table style="border: none;"> <tr> <td style="text-align: center;">♠ ♠ ♠</td> <td style="padding-left: 10px;">3</td> <td></td> </tr> <tr> <td style="text-align: center;">♠ ♠ ♠</td> <td style="padding-left: 10px;">3</td> <td></td> </tr> <tr> <td style="text-align: center;">♠ ♠ ♠</td> <td style="padding-left: 10px;">3</td> <td></td> </tr> <tr> <td style="text-align: center;">♠ ♠ ♠</td> <td style="padding-left: 10px;">3</td> <td></td> </tr> </table> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;"> <math>3 \times 4 = 12</math>  <math>4 \times 3 = 12</math> </div> <div style="text-align: center;"> <math>12 \div 3 = 4</math>  <math>12 \div 4 = 3</math> </div> </div>	♠ ♠ ♠	3		♠ ♠ ♠	3		♠ ♠ ♠	3		♠ ♠ ♠	3	
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<p>X / ÷ 10 or 100</p>	<p>700 <math>\longrightarrow</math> 7</p>												
<p>84 ÷ 7</p>	<p><i>not partitioned into 80 + 4 but into a multiple of 10 which is an exact multiple .....</i></p> 												
<p>84 ÷ 7</p> <p>partitioning</p>	<p><i>not partitioned into 80 + 4 but into a multiple of 10 which is an exact multiple.</i></p> $= (70 + 14) \div 7$ $= (70 \div 7) + (14 \div 7)$ $= 10 + 2$ $= 12$ <p style="text-align: right;"><i>not necessarily recorded this way</i></p>												
<p>600 ÷ 15</p> <p>Factorising</p> <p>This method is not useful unless pupils have good grasp of factors. Can confuse so use with care</p>	$= 600 \div 3 \div 5$ $= 200 \div 5$ $= 40$												

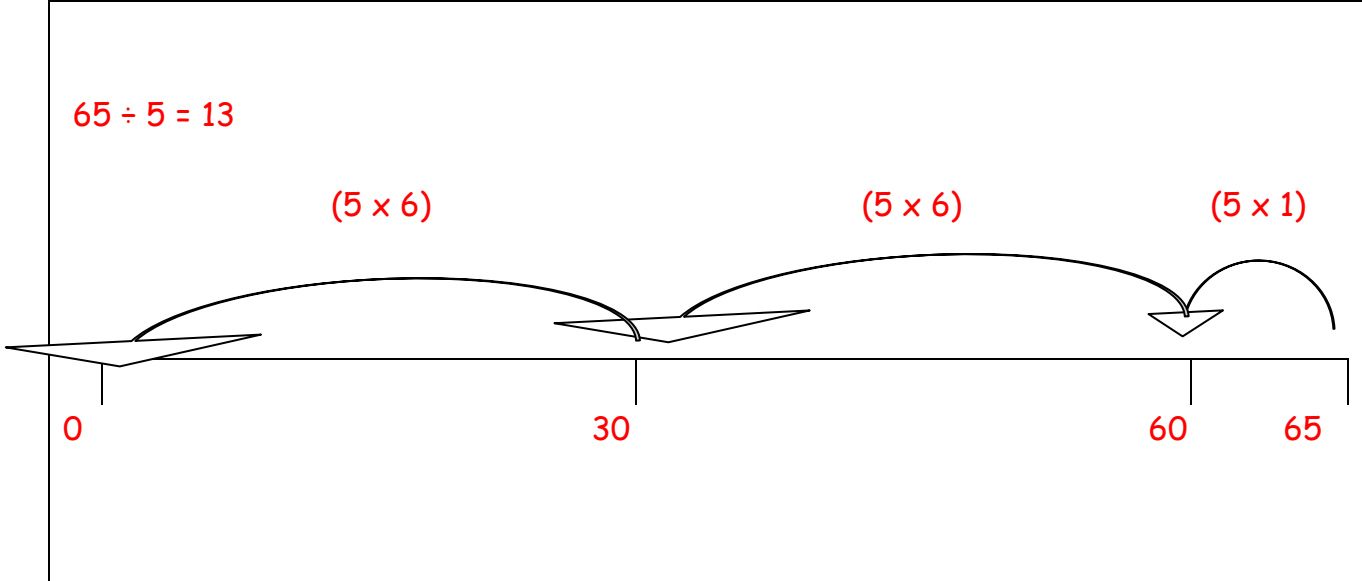
**Stage 3**

Children will begin to recognise division as repeated subtraction. Once mastered both stages 2 and 3 can be repeated using calculations that involve remainders.

<p><math>12 \div 3</math> Chunking / Repeated subtraction</p>	 <p>0      3      6      9      12</p> <p>Can use a numbered or blank line.</p>
<p><math>24 \div 4</math></p>	<p><math>24 - 4 = 20</math> <math>20 - 4 = 16</math> <math>16 - 4 = 12</math> <math>12 - 4 = 8</math> <math>8 - 4 = 4</math> <math>4 - 4 = 0</math>      = 6 (lots of 4 subtracted)</p>

**Stage 4**

Repeated subtraction on a number line can be used with larger numbers by taking away more than one group at a time.





**Stage 6**

In readiness for year 7, more able year six children will be introduced to standard written methods of long division, initially using the chunking strategy, but may look at other written methods (see E.g. 2).

$$972 \div 36$$

$$\begin{array}{r}
 27 \\
 36 \overline{) 972} \\
 \underline{- 720} \quad (20 \times 36) \\
 252 \quad (7 \times 36) \\
 \underline{- 252} \\
 0
 \end{array}$$

$$= 20 + 7 = 27$$

$$972 \div 36 = 27$$

At this stage remainders may now be divided further leading to a decimal answer.

E.g. 2       $634 \div 5$

$$\begin{array}{r}
 126.8 \\
 5 \overline{) 634.0}
 \end{array}$$